Competitive Experimentation in New Markets (Preliminary Version)

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Abstract

Firms can engage in experimentation when they face uncertainty about market demand. Experimentation trades off myopic profit for an increase in future knowledge about market demand, which in turn results in higher expected future profits.

When competitors enter a market, experimentation by first movers may impact the entry choice of second movers. We use a two period model of uncertain Cournot duopoly to explore the impact of endogenous entry on first mover experimentation. We then extend our model to the case of firm actions in other markets that impact knowledge in the incumbent's market. Our two period model is equivalent to a two-dimensional two-armed-bandit problem.

Our results show that experimentation can mitigate the impact of entry in a single market scenario. In a two market scenario, our results show that a strategic effect exists in experimentation of both the incumbent and the entrant.

Our research is motivated by early stage startup firms. Such firms exhibit experimentation behavior, yet is many markets seem to suffer from a second mover advantage phenomenon.

1 Introduction

In repeated market scenarios, uncertainty about market structure and underlaying demand requires firms to consider the impact of current actions on their future knowledge. Choosing actions wisely, firms can experiment in early periods to improve their knowledge about the market in future periods. Improved knowledge is then used to maximize profits in later periods.

A common example for constant experimentation are startup firms. High-tech startups have been a leading force in U.S. Economics for the past 40 years. Recent studies such as NVCA (2011) estimate the impact of venture backed startups at creating 11% of all private sector U.S. jobs and accounting for 10%, or US $3.1 Trillion, of all U.S. annual sales. Startup firms are typically characterized as small, new firms, operating in risky and uncertain markets. These firms exploit advanced technology to "disrupt" existing markets or "create" new ones. Another prominent characteristic of startups are strong resource constraints and a fast pace of changes in the environment in which they operate.

Startup practitioners[1] emphasize the role of experimentation a startup has in its route to success[2]. In these views, a startup differs from others in its ability to experiment efficiently in uncertain environments. The mode of operation is called "pivoting", where startups attempt to find the optimal match between their business model and their market environment through a search process.

Indeed, when observing the startup world, firms exhibit experimentation. In a recent survey of over 600 early stage firms, the average number of pivots

[1] Entrepreneurs, Thought-leaders and Investors

[2] See, *e.g*, "Fail Often, Fail Well" - http://www.economist.com/node/18557776 and "Business plan not working? Time to pivot" - http://venturebeat.com/2010/04/14/business-plan-not-working-time-to-pivot/

reported was above 1 in every 12 month period, and above 3 for successful firms[3].

Since experimentation teaches firms about uncertain market characteristics, first movers can be seen as providing "free information" to potential second movers. Specifically, when the option of entry exists in a market, it is not clear which is better - enter early and experiment or enter late and lose a first mover advantage. Anecdotal evidence of recent startup successes such as Google (Search), facebook (Social Networking), Zynga (Online Gaming) and Groupon (Group buying), indicates that a strong second mover advantage may be in play in uncertain markets.

In light of these phenomena, we focus on how experimentation and endogenous entry decision interact in uncertain markets. We develop a two-period model of uncertain Cournot competition. 2 firms compete through quantities they produce, and observe the realized market prices in every period. The uncertainty in our model stems from two sources: unknown model parameters and temporal shocks. The parameters of the model are determined once by nature and are unknown to the firms in our model, who are aware only of their prior distribution. Temporal shocks to market demand are realized independently in each period. These shocks cause observed prices to produce information about the demand characteristics, but not fully reveal them.

Our focus on observable firm actions and market prices are an attempt to get close to reality as possible. In our view, uncertainty by firms often results from stochastic consumer behavior, and less from unobserved actions or asymmetric information of competitor firms. This is especially true in new markets where startups play in.

[3]Startup Genome Report - http://startupgenome.cc/. A pivot is defined as a "major change in product or business model".

We use a single market scenario in Section 3 to explore the model properties, and determine the direction of experimentation by firm 1 when firm 2 can enter the market in the second period. Our results show that firm 1 can mitigate the impact of entry by firm 2, yet not avoid it completely.

In a two market scenario, we show in Section 4 that experimentation by firm 2 has strategic impact on firm 1's market in future periods. This results in firm 2 manipulating its experimentation to capitalize on the potential to enter firm 1's market. In essence, we are able to show a phenomenon similar to signal jamming as in Riordan (1985) and Mirman et al. (1993a). Our model, however, does not have asymmetric exogenous signals or unobservable actions driving the results.

This result might explain the anecdotal evidence of second mover advantage that exists in many new markets tapped by startups.

Another contribution of our paper is that our model is analogous to a multi-armed bandit model of experimentation. Our contribution in this area is an analysis of a two-dimensional multi-armed scenario, which is a form of a stochastic game.

The research on experimentation (sometimes called learning) is wide and varied. A short review appears in Section 2, and its relevance to our research is described.

Section 5 discusses the results and introduces the process of market education as the final extension to our model. This extension is left for future work.

2 Related Literature and Contribution

Our paper deals with learning, or experimentation, by firms. A requirement for learning to occur, or to be needed, is that the environment faced by the

firms posses some inherent uncertainty. Several types of uncertainty appear in the literature. The first type is about the state of the market, which may be static or dynamic. The second type is about the realization of profits and observations in the market, given firms' actions and the current, unobservable state. A third type deals with uncertainty about competitors, their actions and abilities.

When the environment contains uncertainty of the first and second types, the firm's decision problem can be typically modeled as a multi-armed bandit problem, where a firm faces a set of actions, each yielding a specific, unknown return. In every period in time, the firm chooses an action that faces a tradeoff between exploitation of current knowledge about the expected return from each action, and exploration of uncertain returns to maximize future profits. An excellent introduction to this problem appears in DeGroot (1970), while a modern treatment of solution methods is detailed in Gittins et al. (2011). Rothschild (1974) is credited as one of the early works linking pricing strategies of a firm with the multi-armed bandit problem.

The multi-armed bandit problem is a specific instance of a Markov Decision Process (MDP). See Puterman (2005) for an overview. When observations about the state of nature are stochastic, the process is called a partially observable Markov decision process (POMDP). Astrom (1965) shows that a discrete POMDP is equivalent to an MDP with a continuous belief space.

The generalization of a POMDP to a scenario with multiple decision makers is an instance of a stochastic game, introduced in Shapley (1953) and fully studied in Neyman and Sorin (2003). In stochastic games, the payoffs of players, as well as their observations depend on the actions of competitors. This adds another strategic layer to player behaviors, as they

need to take into account the beliefs of other players, as well as what their actions imply about their learning. The area of stochastic games has been mostly advanced in the mathematics literature, with little economic applications until recently. Some applications in the machine learning and multi-agent reinforcement learning literature exist, such as Hansen et al. (2004) and subsequent literature.

Recently, a stream of Economics literature looks at multi-agent problem with uncertain payoff structures. Works such as Wiseman (2005) and Fudenberg and Yamamoto (2010, 2011) focus on developing folk theorems for stochastic games of the type we are interested in. The theorems apply to infinitely repeated games, and do not analyze specific strategies that may arrive at different possible equilibria.

Our work is closest to traditional applications of learning theory that focus on single market applications with either finite or infinite time periods. Works such as Easley and Kiefer (1989), Aghion et al. (1991), Mirman et al. (1993b), Creane (1994), Trefler (1993) and others focus on monopoly experimentation in a single uncertain market, while works such as Mirman et al. (1994) and Dolores Alepuz and Urbano (1999) analyze the duopoly case. It is interesting to note that most works on duopoly learning analyze the a static (single period game), where players receive ex-ante private signals. Examples of such works include Novshek and Sonnenschein (1982), Gal-Or (1985), Mirman et al. (1993a), Shapiro (1986), Vives (1984). In these works, it is shown that sharing private information among competitors can lead to increased social welfare. In addition, the phenomenon of "signal jamming" where oligopolists manipulate their actions to deter competitors from learning about the true market condition are analyzed.

Our work looks at a finite repeated game where multiple players can en-

ter the market. We focus on finite games because startups typically play in finite markets. Our second focus is on common market beliefs. The majority of work introduces asymmetric information among firms to elicit specific behaviors. Since our work focuses on new markets, we believe common signals are more representing of reality in this case. We do allow, however, for firms to generate endogenous signals through market experimentation. Our contribution is in the analysis of the effects of competitor actions on endogenous learning. Finally, we assume firm actions in markets with experimentation are fully observable. Partial observation is also a possibility in competitive markets, though fully observable actions are probable when firms experiment openly in new markets.

On the topic of startups specifically, the literature is focused mostly on the financing or organizational aspects of the firm. On the finance side, a link between funding and startup success is being researched. As an example, Gompers and Lerner (2006) document findings on the venture capital industry. On the organizational side, a myriad of papers look at the personal and sociological phenomena behind entrepreneurs. Both of these streams of literature try to explain the variation in startup performance based solely on the organizational and funding factors. There is very little focus on the actual business of the firm, and its business environment. One specific example is Zott and Amit (2008) that have introduced a clear distinction between the actual business model and a product strategy of the firm. Their model is empirically tested on firm that have performed an IPO in Europe. While this is a promising direction, their model takes a static view of the firm, as well as analyzes firm performance in hindsight. This ignores the dynamic approach that current entrepreneurs and investors preach, and does not provide an explanation to how firms reach a fit between their business models,

product strategy and the market itself. A good overview of this stream of literature can be found in George and Bock (2011).

3 A Model of Single Market Experimentation

This section introduces a basic single market model where the state of demand and realized prices are uncertain. We later extend this model to contain two markets, and thus present the notation in a general form that will fit the later exposition. Our model[4] consists of a single market indexed $j = 1$ and two time periods, indexed $t \in \{1, 2\}$. Our market has two firms, indexed $i \in \{1, 2\}$. In each period, the firms in the market choose quantities of a homogeneous product to produce. Each firm's production decision is denoted q_{ijt} for firm i in market j at time t. The total aggregated production is denoted $Q_{jt} = q_{1jt} + q_{2jt}$. Since in this section we only focus on one market, we use Q_t to denote the total production in each time period. The production decision of all firms in the market are common knowledge. After production decisions are made, demand is realized and prices are observed by all players. The prices are determined by an inverse demand function, $g(\cdot, \cdot)$, and a noise parameter, ε_t:

$$P_t = g(\gamma, Q_t) + \varepsilon_t \tag{1}$$

We assume ε_t is drawn i.i.d from a density f_ε which exhibits the monotone likelihood ratio property (MLRP) and has expectation 0.

The parameter γ characterizes the type of the market. There are two possible market scenarios, a "good" market with parameter $\overline{\gamma}$ and a "bad" market with parameter $\underline{\gamma}$. We define an order on γ w.l.o.g such that $\overline{\gamma} > \underline{\gamma}$. The market type is unknown to the firms in the market. Prior to the first

[4]The notation is based on Mirman et al. (1993b, 1994) and subsequent papers.

period, the market type is determined by nature randomly. The probability of the market being of the "good" type is ρ_0. This prior probability is common knowledge, as well as the distribution f_ε. To have meaningful results, we assume possible price realizations always overlap between both market types, so that a firm cannot infer the market type from a single observation of P_t. That is, for all realizations of $P_t(Q_t)$ we have $f_\varepsilon(P_t - g(\gamma, Q_t)) \neq 0$ for all $\gamma \in \{\overline{\gamma}, \underline{\gamma}\}$.

We assume the following structure on the inverse demand functions:

Assumption 1 *1. For each quantity Q the expected prices are higher under good demand: $g(\overline{\gamma}, Q) > g(\underline{\gamma}, Q)$*

2. $g(\cdot, \cdot)$ exhibits increasing differences[5] in γ and Q:

$$g'(\overline{\gamma}, Q) = \frac{dg(\overline{\gamma}, Q)}{dQ} > \frac{dg(\underline{\gamma}, Q)}{dQ} = g'(\underline{\gamma}, Q) \tag{2}$$

The implications of Assumption 1 will be made clear later in this section.

Our market has two potential firms in it. Their marginal costs of production are fixed, equal and normalized to zero. Firm 1, called the *incumbent*, enters the market in period 1, pays a fixed entry cost c and produces quantity $Q_1 = q_{111}$. Firm 2, called the *entrant*, does not produce anything in period 1. It observes the production decision of firm 1 in period 1, and the realized price P_1.

The realized quantity and price induce a new common belief about the state of the market in period 2. This belief is generated using Bayes rule, and we denote it as

$$\rho = \rho(P_1|Q_1) = \frac{\rho_0 f(P_1 - g(\overline{\gamma}, Q_1))}{\rho_0 f(P_1 - g(\overline{\gamma}, Q_1)) + (1 - \rho_0) f(P_1 - g(\underline{\gamma}, Q_1))} \tag{3}$$

[5]Note that γ can achieve only two values.

The belief, ρ, is a function of Q_1 and the realized price P_1. Since P_1 is a random variable, so is ρ.

Given this belief firm 2 decides whether to enter the market. If firm 2 enters the market, it pays a fixed cost c, and produces q_{212}. Firm 1 observes firm 2's entry decision, and then produces q_{112} simultaneously with firm 2. Profits are discounted with a discount factor of δ for each period. Both firms maximize their individual sum of profits from the production in both periods (if they produce). We assume $\delta = 1$ in the majority of the paper, unless otherwise noted, as it does not affect the results significantly. The expected profit in each period is therefore:

$$\pi_{1jt}(q_{1jt}, q_{2jt}, \rho) = q_{1jt} \cdot (\rho \cdot g(\overline{\gamma}, Q_{jt}) + (1 - \rho) \cdot g(\underline{\gamma}, Q_{jt})) \tag{4}$$

For clarification, firm 1's profit function in period 1 is $\pi_{111}(Q_1, 0, \rho_0)$. When firm 2 does *not* enter in period 2, firm 1's profit in period 2 is $\pi_{112}(q_{112}, 0, \rho)$ and when firm 2 does enter, the profit is $\pi_{112}(q_{112}, q_{212}, \rho)$.

Firm 2's profit, when it decides to enter the market is therefore

$$\pi_{212}(q_{112}, q_{212}, \rho) = \pi_{112}(q_{112}, q_{212}, \rho) - c \tag{5}$$

We are interested in firm 1's choice of the first period quantity $q_{111} = Q_1$ as well as firm 2's entry behavior. Our approach is to analyze the subgame perfect (Beyesian) equilibrium resulting from this game. Therefore, we first begin with an analysis of the second period.

We assume that the regularity conditions required for an interior unique equilibrium in each period hold for both the monopoly and duopoly cases[6]. Specifically, decreasing marginal revenues is a sufficient condition in our

[6] A proof for the existence of equilibrium and its uniqueness in the duopoly case can be found in Mirman et al. (1994).

case. We note that such an equilibrium might be in mixed strategies over the quantities in the second period. As those quantities are not the main interest of this paper, we do not focus on them. We let q^*_{112} and q^*_{212} be the second period equilibrium productions of both firms.

We define the value function as the maximal equilibrium profit that can be achieved by firm 1 in period 2 given its belief ρ:

$$V(\rho) = \mathbb{E}_\varepsilon[\pi_{112}(q^*_{112}, q^*_{212}, \rho)] \tag{6}$$

To simplify notation, we denote the monopoly value as $V^m(\rho)$ and the duopoly value, *i.e.* the value when firm 2 enters in period 2, as $V^d(\rho)$. Because of the symmetric nature of the firms, the value achieved by firm 2 is $V^d(\rho) - c$ if it enters, and zero otherwise.

Firm 1's optimization problem is therefore

$$\max_{Q1} \pi_{111}(Q_1, 0, \rho_0) + \delta \cdot \mathbb{E}_{P_1}[V(\rho(P_1|Q_1))] \tag{7}$$

We assume this optimization problem has the conditions for a unique maximum. Under linear demand and all further examples in this paper, these conditions hold. When firm 1 is myopic ($\delta = 0$), experimenting will not impact its second period expected profits. As a result, firm 1 will produce the optimal single period quantity which we call the *no experimentation* quantity, denoted Q_1^{NE}.

We now turn to analyze the cases where firm 1 deviates from producing Q_1^{NE} in period 1, and the direction of this deviation. Throughout the rest of the paper, we assume that $\delta = 1$, unless otherwise noted.

3.1 Incentives for Experimentation and The Value of Information

For a value function $V(\rho)$ information is *valuable* when $\mathbb{E}[V(\rho)] > V(\mathbb{E}[\rho]) = V(\rho_0)$. By Jensen's inequality, $V''(\rho) > 0$ is a sufficient condition for information to be valuable. It is important to note that not all *useful* information is valuable. Information is useful when the optimal second period quantities change as a result of changes in ρ. Formally, information is useful when $\frac{\partial q^*_{i12}}{\partial \rho} \neq 0$. This might not mean, however, that the higher value (profit) achieved in the second period with more information will offset the loss of profit from experimenting in period 1.

As an example of non-valuable information, assume a linear demand model where $g(\overline{\gamma}, q) = \overline{a} - b \cdot q$ and that $g(\underline{\gamma}, q) = \underline{a} - b \cdot q$ with $\overline{a} > \underline{a}$. This demand violates Assumption 1. In this case, when observing the price P_1, the expected price difference between the "good" and "bad" markets does not depend on the choice of Q_1. Thus, although different realizations of ρ will change the optimal quantity produced, changing Q_1 from its myopic optimum is not necessary, as staying at this optimum will yield the same amount of learning.

When firm 1 remains a monopoly in period 2, an envelope argument shows that the value function $V^m(\rho)$ is strictly convex (see De la Fuente (2000) pp. 314 and Nyarko (1994)). As a result, information is always valuable for a monopoly in the second period if it is useful.

The case for a duopoly is less clear. For certain demand functions, it is possible that the resulting value function will be linear or concave[7]. In that situation, firm 1 is better off not experimenting if it expects firm 2 to enter

[7]Dolores Alepuz and Urbano (1999) show that under normally distributed noise and normal prior on the quantity coefficient in linear demand, the value function is strictly convex.

in the second period.

We restrict our analysis to cases where information is valuable for a duopoly as well, that is when $(V^d)''(\rho) > 0$. The remaining cases are of extreme interest for future research, as they may yield very different trade-offs between experimentation and information leaking.

Given assumption 1 and our assumption on the duopoly value function, we prove the following lemma:

Lemma 1 *Let* $g(\overline{\gamma}, q) = \overline{a} - \overline{b} \cdot q^{\theta}$, $g(\underline{\gamma}, q) = \underline{a} - \underline{b} \cdot q^{\theta}$, *with* $\theta > 0$ *and* $\overline{a}/\overline{b} \neq \underline{a}/\underline{b}$. *Under Assumption 1 (*i.e. $\overline{a} > \underline{a}$ *and* $\overline{b} < \underline{b}$*)*

$$Q_1^{NE} < Q_1^d < Q_1^m \tag{8}$$

Lemma 1 states that when demand is structured as defined, firm 1's optimal quantity in period 1 is larger than the no experimentation quantity. If firm 1 expects firm 2 to enter, it will produce Q_1^d which is smaller than Q_1^m. We note that the case of $\theta = 1$ is the case of linear demand, and as such our lemma covers both concave inverse demand functions as well as slightly convex ones.

When Assumption 1 holds, it can be shown that Lemma 1 is correct for any demand structure that has $0 < (V^d)''(\rho) < (V^m)''(\rho)$.

We now turn to analyze the entry decision of firm 2. When the entry cost c is lower than the expected duopoly value $V^d(\rho)$, firm 2 will surely enter the market. On the other hand, if for every value of ρ, $c > V^d(\rho)$, firm 2 will not enter the market.

The interesting case is when for some values of ρ, $c > V^d(\rho)$, while for others, $c < V^d(\rho)$. To simplify the analysis, we assume firm 1 always enters

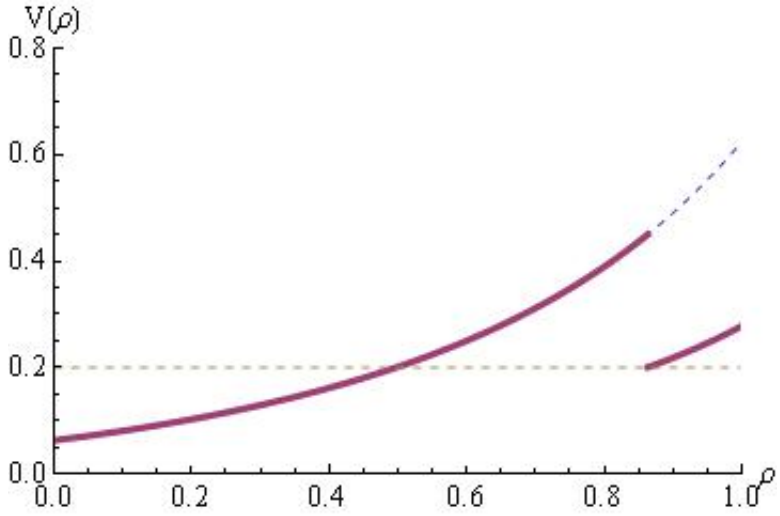


Figure 1: Expected profit for firm 1 under different beliefs and linear demands. ($\overline{a} = 1, \overline{b} = 0.4, \underline{a} = 0.5, \underline{b} = 1, c = 0.2$).

the market[8] in period 1, *i.e.* that $\pi_{111}(Q_1, 0, \rho_0) + \delta \cdot \mathbb{E}_{P_1}[V^d(\rho(P_1|Q_1))] > c$.

Under Assumption 1, firms profits under both monopoly and duopoly are higher under "good" demand than under "bad" demand. This implies that $V(1) > V(0)$. When firm 2 enters the market, the expected profit for firm 1 drops. Figure 1 shows the impact of entry cost on the expected second period profit for firm 1.

The next proposition characterizes the quantity firm 1 will produce in period 1 for different values of the entry cost, c.

Proposition 1 *Under Assumption 1:*

1. *When* $V^d(0) > c$ *and* $(V^d)''(\rho) > 0$ *firm* 1 *will produce* $Q_1 = Q_1^d > Q_1^{NE}$, *and firm* 2 *will always enter the market.*

[8] When this assumption does not hold for all values of Q_1, the results remain similar, yet require more technicalities.

2. *When* $V^d(1) < c$*, firm* 1 *will produce* $Q_1 = Q_1^m > Q_1^{NE}$*, and firm* 2 *will never enter the market.*

3. *When* $V^d(0) < c < V^d(1)$ *and* $(V^m)''(\rho) > (V^d)''(\rho) > 0$*, firm* 1 *will produce* $Q_1^{NE} < Q_1^d < Q_1 < Q_1^m$*.*

Proposition 1 shows that uncertainty about the market state will cause firm 1 to credibly believe firm 2 will enter the market even when the underlying demand is of the "bad" type. As a result, firm 1 will manipulate its first period quantity, Q_1, to mitigate this risk. We do see, however, that the expected second period profit for firm 1 is above the duopoly profit for conditions 2 and 3 of Proposition 1. This can explain why firms "rush" to a market. A first mover advantage exists in these markets for such firms.

When the inverse demand functions are as specified in Lemma 1, the condition $(V^m)''(\rho) > (V^d)''(\rho) > 0$ always holds. The meaning is that item 3 of Proposition 1 will hold when entry costs are high enough, but not too high. As a result we expect to see firms producing quantities below the optimal monopoly quantity in period 1, yet above the no experimentation quantity.

4 Strategic Learning in Multiple Markets

In this section, we extend our model to a two market scenario. The extension is done in our context of startups. It is common for founders of startups to have a basic plan for their initial business model and a market in which they intend to compete. We therefore add market no. 2 to our model, yielding $j \in \{1, 2\}$. We assume only one market can be a "good" market, while the other is a "bad" market. Thus, prior to the game's beginning, nature can choose among two states. In the first state, market 1 is the "good" market,

i.e. $(\gamma_1, \gamma_2) = (\overline{\gamma}, \underline{\gamma})$ and in state 2, $(\gamma_1, \gamma_2) = (\underline{\gamma}, \overline{\gamma})$.

The prior probability, ρ_0, is now the probability of state 1.

In our extended model, we assume firms can only participate in one of the two markets in each period. This is consistent with startups that focus on a single product in every specific timeframe. Thus, every firm produces a quantity $q_{ij1} > 0$ in period 1 only in one market. After observing the quantities produced by competitors and resulting realized prices P_{11} and P_{21}, firms update their beliefs about the state of the world, ρ. To simplify notation, we define:

$$\begin{aligned} \overline{g}_1 &= g(\overline{\gamma}, Q_1) & \underline{g}_1 &= g(\underline{\gamma}, Q_1) \\ \overline{g}_2 &= g(\overline{\gamma}, Q_2) & \underline{g}_2 &= g(\underline{\gamma}, Q_2) \\ \overline{f}_1 &= f(P_1 - g(\overline{\gamma}, Q_1)) & \underline{f}_1 &= f(P_1 - g(\underline{\gamma}, Q_1)) \\ \overline{f}_2 &= f(P_2 - g(\overline{\gamma}, Q_2)) & \underline{f}_2 &= f(P_2 - g(\underline{\gamma}, Q_2)) \end{aligned}$$

When Q_1 and Q_2 are the total productions in markets 1 and 2 respectively in period 1, and P_1 and P_2 are the realized prices in those markets.

We thus have:

$$\rho = \rho(P_1, P_2 | Q_1, Q_2) = \frac{\rho_0 \overline{f}_1 \underline{f}_2}{\rho_0 \overline{f}_1 \underline{f}_2 + (1 - \rho_0) \underline{f}_1 \overline{f}_2} \tag{9}$$

We note that the updated belief now depends on the outcome in both markets.

After observing ρ, each firm may choose to switch the market it plays in and pay a switching cost c, or remain in the same market. After the decision to switch or remain, which is public, each firm produces a second period quantity q_{ij2}. This game has multiple equilibria because of its symmetric nature. We therefore focus on a specific case of interest - we limit our

analysis to the case where both firms choose different markets in period 1, and in which firm 1 does not switch to market 2 after observing ρ.

We define the value functions V and W for market 1 and 2 respectively as follows. As firm 1 does not compete in market 2, its production in market 2, q_{12t}, is always zero.

$$V_i(\rho) = \mathbb{E}_\varepsilon[\pi_{i12}(q^*_{112}, q^*_{212}, \rho)] - c \cdot \mathbb{I}(i = 2) \tag{10}$$

$$W_i(\rho) = \mathbb{E}_\varepsilon[\pi_{i22}(0, q^*_{222}, \rho)] \tag{11}$$

The following two lemmas tell us how the expected value in the second period changes with the production choices in the first period:

Lemma 2

$$\frac{\partial \mathbb{E}_{P_1,P_2}[V_1(\rho(P_1, P_2|Q_1, Q_2))]}{\partial Q_1} = \left[\overline{g}'_1 - \underline{g}'_1\right] \int\int V_1''(\rho) \frac{\partial \rho}{\partial P_1} \rho_0(1-\rho) \overline{f}_1 \underline{f}_2 dP_1 dP_2 \tag{12}$$

Lemma 3

$$\frac{\partial \rho}{\partial P_1} > 0, \quad \frac{\partial \rho}{\partial P_2} < 0 \tag{13}$$

These two lemmas allow us to forecast the direction of change in non-experimentation quantity each firm will take. The intuition is that each firm plays in a single market in period 1 and thus its actions have a direct effect on the beliefs for its market. When the belief for one market increases, however, it immediately decreases for the other market. Since $W(\rho) = V(1-\rho)$ the expected profit from market 2 declines when the expected profit in market 1 increases.

Suppose both firms have chosen to produce Q_1 and Q_2 in period 1. After observing the updated belief, ρ, firm 2 will switch to market 1 only if the

duopoly profit in market 1 is higher than the expected monopoly profit in market 2. In order for this to happen, it is required that there exist a value of ρ which we call ρ_c above which $V^d(\rho_c) - c > W^m(\rho)$. If this condition does not hold, firm 2 will never switch, and firms 1 and 2 will produce Q_1^m and Q_2^m.

The following proposition describes the conditions and optimal quantities Q_1 and Q_2 firms 1 and 2 will produce under linear demand and uniform error distribution:

Proposition 2 *Let* $g(\overline{\gamma}, q) = \overline{a} - \overline{b} \cdot q$, $g(\underline{\gamma}, q) = \underline{a} - \underline{b} \cdot q$, *and* $\overline{a}/\overline{b} \neq \underline{a}/\underline{b}$. *Let Assumption 1 hold and assume* ε *is distributed uniformly between* $[-1/2, 1/2]$.

1. *If* $W^m(\rho) > V^d(\rho) - c$ *for all* ρ, ρ_c *does not exist, and firm* 2 *will never switch. Both firms will produce* Q_1^m *and* Q_2^m.

2. *If* ρ_c *exists (*i.e. *for some* ρ, $c < V^d(\rho) - W^m(\rho)$*), firm* 1 *is expected to produce less than* Q_1^m *and firm* 2 *above* Q_2^m.

Proposition 2 shows that introducing the threat of entry changes the behavior of the firms dramatically in period 1. Specifically, a firm which is threatened by a competitor, will lower its produced quantity from the monopoly amount. Firm 2, however, will overproduce.

The intuition is that firm 1 would like to deter entry of firm 2 by learning less in its own market. Producing less quantities lowers the separation between both demand curves and produces a less informative signal. Firm 2 would like to increase its quantity to increase this separation in its own market, and also strategically reverse the effect of firm 1's actions in market 1.

One thing to note is that in the single market case, a high enough entry cost $c > V^d(\rho)$ was required to let firm 1 mitigate the loss from entry of firm 2. In the two market scenario, even with zero entry cost, it is enough that market 2 is lucrative enough as a monopoly compared to a duopoly in market 2 to elicit the same effect. In the case of linear demand, it is enough that $V^m(0) > \frac{4}{9}V^d(0.5)$ for firm 2 to never switch to market 1. This happens when

$$\frac{\underline{a}}{0.5\overline{a} + 0.5\underline{a}} > \frac{2}{3}\sqrt{\frac{\underline{b}}{0.5\overline{b} + 0.5\underline{b}}} \tag{14}$$

When this condition does not hold, firm 2 will switch to market 1 given a high realization of ρ, and firm 1 will lower its production to mitigate this effect.

5 Conclusion

This paper looks at how optimal experimentation changes when firms can choose to be a first mover or a second mover in a market. Our model compares the conditions for optimal experimentation of monopolies and duopolies. We find that having firms compete in two markets changes their experimentation behavior compared to a single market scenario. The reason is not because of the change in the value of alternative option facing the second mover, but rather because the second mover's history affects the knowledge of both players. When compared to the signal jamming literature, we receive a similar effect. The information structure facing the firms in our model, however, is different than standard signaling games, as the firms themselves have no private information.

Our model is also limited to convex value functions and inverse demand functions with increasing differences. In other works it was shown that

weaker conditions on demand structure can be established under which the value function is convex. We conjecture that weaker conditions are sufficient for establishing our results, and we will explore this direction in future work.

Finally, it is sometimes claimed that innovative firms in new markets "create" their market. In the context of our model, market creation will appear as changes in the demand functions throughout time periods. The changes will be determined by firm actions, which will create state dependence of future knowledge through the history of actions. Firms will then have a tradeoff between experimentation and strategic market education. We believe this is a promising direction for future research.

6 Appendix

PROOF OF LEMMA 1: Assign $\hat{a} = \rho\overline{a} + (1-\rho)\underline{a}$ and $\hat{b} = \rho\overline{b} + (1-\rho)\underline{b}$.

We calculate $V^m(\rho)$ and $V^d(\rho)$ directly:

$$V^m(\rho) = \hat{a}^{(\theta+1)/\theta}\ \hat{b}^{-(1/\theta)}\ \theta\ (1+\theta)^{(-1-\theta)/\theta} \tag{15}$$

$$V^d(\rho) = \hat{a}^{(\theta+1)/\theta}\ \hat{b}^{-(1/\theta)}\ \theta\ (2+\theta)^{(-1-\theta)/\theta}\ 2^{(1-\theta)/\theta} \tag{16}$$

It is easy to verify that $(V^m)''(\rho) > 0$, and since

$$V^d(\rho) = \frac{(2+\theta)^{(-1-\theta)/\theta}\ 2^{(1-\theta)/\theta}}{(1+\theta)^{(-1-\theta)/\theta}} V^m(\rho) < V^m(\rho) \tag{17}$$

we can conclude that

$$0 < (V^d)''(\rho) < (V^m)''(\rho) \tag{18}$$

Lemma 2 of Mirman et al. (1993b), which also holds in Mirman et al. (1994) shows that

$$\frac{d\mathbb{E}_P V(\rho(P,Q))}{dQ} = (\overline{g}' - \underline{g}') \int V''(\rho)(1-\rho)\rho_0 \frac{d\rho}{dP} \overline{f} dP \tag{19}$$

When $\overline{g} = g(\overline{\gamma}, Q)$, $\underline{g} = g(\underline{\gamma}, Q)$ and $\overline{f} = f(P - g(\overline{\gamma}, Q))$.

Assumption 1 yields $(\overline{g}' - \underline{g}') > 0$ while the MLRP of f_ε yields $\frac{d\rho}{dP} > 0$

We therefore have:

$$0 < \frac{d\mathbb{E}_P V^d(\rho(P,Q))}{dQ} < \frac{d\mathbb{E}_P V^m(\rho(P,Q))}{dQ} \tag{20}$$

The fact that the second period first order is non-negative implies that firm 1 will produce higher quantities than Q^{NE} when solving its optimization problem in (7).

In addition, the deviation from optimality is smaller when firm 1 is facing a duopoly. As a result, $Q_1^d < Q_1^m$, as expected.

Proof of Proposition 1:

1. Given Assumption 1, $V^d(0) > c$ implies $V^d(\rho) > c$. Therefore, firm 2 will always enter the market in period 2, and as a result, firm 1 will produce the optimal duopoly quantity, $Q_1^d > Q_1^{NE}$, anticipating that entry.

2. Given Assumption 1, $V^d(1) < c$ implies $V^d(\rho) < c$. Therefore, firm 2 will never enter the market in period 2, and as a result, firm 1 will produce the optimal monopoly quantity, Q_1^m, which is larger than Q_1^{NE} as shown in Mirman et al. (1993b).

3. We define $\rho_c = \{\rho | V^d(\rho) < c\}$. These are the values of ρ for which firm 2 will not enter the market in period 2, if ρ is realized. By Lemma 2 of Mirman et al. (1993b),

$$\frac{d\mathbb{E}_P V(\rho(P,Q))}{dQ} = (\overline{g}' - \underline{g}') \int V''(\rho)(1-\rho)\rho_0 \frac{d\rho}{dP} \overline{f} dP \tag{21}$$

which can be rewritten as

$$\frac{d\mathbb{E}_P V(\rho(P,Q))}{dQ} = (\overline{g}' - \underline{g}') \int_{\rho_c} (V^m)''(\rho)(1-\rho)\rho_0 \frac{d\rho}{dP} \overline{f} dP + \tag{22}$$

$$(\overline{g}' - \underline{g}') \int_{\rho \notin \rho_c} (V^d)''(\rho)(1-\rho)\rho_0 \frac{d\rho}{dP} \overline{f} dP \tag{23}$$

Thus

$$\frac{d\mathbb{E}_P V^d(\rho(P,Q))}{dQ} < \frac{d\mathbb{E}_P V(\rho(P,Q))}{dQ} < \frac{d\mathbb{E}_P V^m(\rho(P,Q))}{dQ} \tag{24}$$

which completes the proof.

Proof of Lemma 2: By definition

$$\mathbb{E}_{P_1,P_2}[V_1(\rho(P_1,P_2|Q_1,Q_2))] = \int\int V_1(\rho) h(P_1,P_2|Q_1,Q_2) dP_1 dP_2 \tag{25}$$

Where

$$h(P_1,P_2|Q_1,Q_2) = \rho_0 \overline{f}_1 \underline{f}_2 + (1-\rho_0) \underline{f}_1 \overline{f}_2 \tag{26}$$

is the pdf of ρ.

Taking the derivative with respect to Q_1 results in:

$$\frac{\partial \mathbb{E}_{P_1,P_2}[V_1(\rho)]}{\partial Q_1} = \int\int \left(V_1'(\rho) \frac{\partial \rho}{\partial Q_1} \cdot h + V_1(\rho) \frac{\partial h}{\partial Q_1} \right) dP_1 dP_2 \tag{27}$$

Since $\rho h = \rho_0 \overline{f}_1 \underline{f}_2$, taking the derivative with respect to P_1 and to Q_1 yields:

$$\frac{\partial \rho}{\partial P_1} = \frac{\rho_0 \overline{f}_1' \underline{f}_2 - \rho \frac{\partial h}{\partial P_1}}{h} \tag{28}$$

$$\frac{\partial \rho}{\partial Q_1} = \frac{-\rho_0 \overline{f}_1' \overline{g}_1' \underline{f}_2 - \rho \frac{\partial h}{\partial Q_1}}{h} \tag{29}$$

Plugging $\frac{\partial h}{\partial P_1} = \rho_0 \overline{f}'_1 \underline{f}_2 + (1-\rho_0)\underline{f}'_1 \overline{f}_2$ into $\frac{\partial h}{\partial Q_1}$ we get

$$\begin{aligned} \frac{\partial h}{\partial Q_1} &= -\rho_0 \overline{f}'_1 \overline{g}'_1 \underline{f}_2 - (1-\rho_0)\underline{f}'_1 \underline{g}'_1 \overline{f}_2 \\ &= -\overline{g}'_1 \frac{\partial h}{\partial P_1} + (1-\rho_0)(\overline{g}'_1 - \underline{g}'_1)\underline{f}'_1 \overline{f}_2 \end{aligned} \tag{30}$$

Inserting $\frac{\partial h}{\partial Q_1}$ and $\frac{\partial h}{\partial P_1}$ into (29) yields

$$\frac{\partial \rho}{\partial Q_1} = -\overline{g}'_1 \frac{\partial \rho}{\partial P_1} - \frac{\rho}{h}(1-\rho_0)(\overline{g}'_1 - \underline{g}'_1)\underline{f}'_1 \overline{f}_2 \tag{31}$$

In addition, plugging $\frac{\partial h}{\partial Q_1}$ into the second part of (27) and integrating by parts results in:

$$\int\int \left(V_1(\rho) \frac{\partial h}{\partial Q_1} \right) dP_1 dP_2 = \tag{32}$$

$$\int\int V'_1(\rho) \frac{\partial \rho}{\partial P_1} \left[\rho_0 \overline{f}_1 \overline{g}_1 \underline{f}_2 + (1-\rho_0)\underline{f}'_1 \underline{g}'_1 \overline{f}_2 \right] dP_1 dP_2 \tag{33}$$

We can therefore use (32) to rewrite (27) as

$$\frac{\partial \mathbb{E}_{P_1,P_2}[V_1(\rho)]}{\partial Q_1} = \tag{34}$$

$$\int\int V'_1(\rho) \frac{\partial \rho}{\partial P_1} \left[-\overline{g}'_1 h + \rho_0 \overline{g}'_1 \overline{f}_1 \underline{f}_2 + (1-\rho_0)\underline{g}'_1 \underline{f}_1 \overline{f}_2 \right] dP_1 dP_2 \tag{35}$$

$$-\int\int V'_1(\rho)\rho(\overline{g}'_1 - \underline{g}'_1)(1-\rho_0)(\underline{f}'_1 \overline{f}_2) dP_1 dP_2 = \tag{36}$$

$$(\overline{g}'_1 - \underline{g}'_1)\int\int V'_1(\rho) \frac{\partial \rho}{\partial P_1}(1-\rho_0)\underline{f}_1 \overline{f}_2 dP_1 dP_2 - \tag{37}$$

$$(\overline{g}'_1 - \underline{g}'_1)\int\int V'_1(\rho)\rho(1-\rho_0)\underline{f}'_1 \overline{f}_2 dP_1 dP_2 = \tag{38}$$

$$(\overline{g}'_1 - \underline{g}'_1)\int\int V''_1(\rho) \frac{\partial \rho}{\partial P_1}\rho(1-\rho_0)\underline{f}_1 \overline{f}_2 dP_1 dP_2 = \tag{39}$$

$$(\overline{g}'_1 - \underline{g}'_1)\int\int V''_1(\rho) \frac{\partial \rho}{\partial P_1}\rho_0(1-\rho)\overline{f}_1 \underline{f}_2 dP_1 dP_2 \tag{40}$$

Reaching (39) is done by integrating by parts (38), and the final transformation is done using the fact that $\rho(1-\rho_0)\underline{f}_1\overline{f}_2 = \rho_0\overline{f}_1\underline{f}_2(1-\rho)$, which finalizes our proof.

PROOF OF LEMMA 3: Deriving directly

$$\frac{\partial\rho}{\partial P_1} = \frac{\rho_0(1-\rho_0)\overline{f}_2\underline{f}_2(\overline{f}'_1\underline{f}_1 - \underline{f}'_1\overline{f}_1)}{(\rho_0\overline{f}_1\underline{f}_2 + (1-\rho_0)\underline{f}_1\overline{f}_2)^2} \tag{41}$$

By the MLRP of f we have $\overline{f}'_1\underline{f}_1 - \underline{f}'_1\overline{f}_1 > 0$ which completes the proof.

The results for other partial derivatives are derived similarly.

PROOF OF PROPOSITION 2: We focus on the case where ρ_c exists.

Define:

$$\hat{a} = \rho_0\overline{a} + (1-\rho_0)\underline{a} \qquad \hat{b} = \rho_0\overline{b} + (1-\rho_0)\underline{b} \tag{42}$$

$$\tilde{a} = (1-\rho_0)\overline{a} + \rho_0\underline{a} \qquad \tilde{b} = (1-\rho_0)\overline{b} + \rho_0\underline{b} \tag{43}$$

$$\hat{v} = \rho_0\frac{\overline{a}^2}{4\overline{b}} + (1-\rho_0)\frac{\underline{a}^2}{4\underline{b}} - \frac{\hat{a}^2}{4\hat{b}} \tag{44}$$

$$\tilde{v} = \rho_0\frac{\overline{a}^2}{4\overline{b}} + (1-\rho_0)\frac{\underline{a}^2}{4\underline{b}} - \frac{\tilde{a}^2}{4\tilde{b}} \tag{45}$$

$$e_1 = (\overline{a} - \overline{a} - (\overline{b} - \underline{b})Q_1 \tag{46}$$

$$e_2 = (\overline{a} - \overline{a} - (\overline{b} - \underline{b})Q_2 \tag{47}$$

e_1 and e_2 are the expected price differences to observe in a single market by each firm.

We directly calculate $\mathbb{E}_{P_1,P_2}[V(Q_1,Q_2)]$ and $\mathbb{E}_{P_1,P_2}[W(Q_1,Q_2)]$:

$$\mathbb{E}_{P_1,P_2}[V^m(Q_1,Q_2)] = \hat{v}(e_1 + e_2 - e_1\cdot e_2) + \frac{\hat{a}^2}{4\hat{b}} \tag{48}$$

$$\mathbb{E}_{P_1,P_2}[V^d((Q_1,Q_2)] = \frac{4}{9}V^m(Q_1,Q_2) \tag{49}$$

$$\mathbb{E}_{P_1,P_2}[W^m(Q_1,Q_2)] = \tilde{v}(e_1 + e_2 - e_1\cdot e_2) + \frac{\tilde{a}^2}{4\tilde{b}} \tag{50}$$

The profit function observed by both players is:

$$\Pi_1(Q_1, Q_2) = \pi_1(Q_1, 0, \rho_0) + \alpha_1 \mathbb{E}_{P_1,P_2}[V^m(Q_1, Q_2)] + \tag{51}$$

$$(1 - \alpha_1)\mathbb{E}_{P_1,P_2}[V^d((Q_1, Q_2)] \tag{52}$$

$$\Pi_2(Q_1, Q_2) = \pi_2(0, Q_2, \rho_0) + \alpha_2 \mathbb{E}_{P_1,P_2}[W^m(Q_1, Q_2)] + \tag{53}$$

$$(1 - \alpha_2)\mathbb{E}_{P_1,P_2}[V^d((Q_1, Q_2)] \tag{54}$$

α_1 and α_2 are determined by the entry cost c and existence of ρ_c. When ρ_c does not exist, $\alpha_1 = \alpha_2 = 1$, and only monopoly profit is expected in the second period. Both α values decrease when c decreases. Thus, both firms will produce Q_i^m when the entry cost c is high.

By changing the variable Q_2 to $\hat{Q}_2 = -Q_2$, both profit functions become supermodular in $(Q_1, \hat{Q}_2)$. In addition, Π_1 has increasing differences in (α_1, Q_1) and Π_2 has increasing differences in $(\alpha_2, \hat{Q}_2)$ when $\overline{b} + \underline{a}\overline{b} - \underline{b}\overline{a} > 0$.

Topkis's theorem then states than $Q_1^m < Q_1$ and $Q_2^m < \hat{Q}_2$, and finishes the proof.

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